

On the geometrical background of Dionysius Thrax' definition of comparatives

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The article analyzes Dionysius Thrax' definition of the comparative, according to which the comparative introduces a comparison of one object with another one of the same kind or of one object with more objects of a different kind; this definition, which is apparently contradicted by the actual linguistic usage of Old Greek, has always presented hermeneutic difficulties to both ancient and contemporary commentators and historians of grammatical thought. None of the explanations offered so far is convincing, as they limit themselves to judge the definition unsound and careless, without trying to read it inside the possible theoretical frame of conceptual reference, within which it might have been formed.

Two hypotheses are proposed. The first is that a definition of the comparative could have been introduced into the realm of technical grammar only after the establishment of a philosophical 'metatheory' of comparison around the controversies which accompanied the definition of 'relation' in the Platonic-Aristotelean tradition. The second is that Dionysius' definition is shaped within a geometrical frame of reference, linking the definition of the comparative to the geometrical, Eudoxean and Euclidean, definition of 'ratio' (*logos*) as "a sort of relation in respect of size between *two* magnitudes of the *same kind*", which closely recalls Dionysius' definition and is mainly based on the restriction (*diorismos*) that the magnitudes forming a ratio must be homogeneous*.

1. Dionysius Thrax' definition of the comparative has always presented hermeneutic difficulties to both ancient and contemporary commentators and historians of grammatical thought.

This definition reads as follows:

Συγκριτικὸν δέ ἐστι τὸ τὴν σύγκρισιν ἔχον ἐνὸς πρὸς ἓνα ὁμοιογενῆ, ὡς Ἀχιλλεύς ἀνδρείοτερος Αἴαντος, ἢ ἐνὸς πρὸς πολλοὺς ἑτερογενεῖς, ὡς Ἀχιλλεύς ἀνδρείοτερος τῶν Τρώων.

"The comparative is the noun [Dionysius does not distinguish between nouns and adjectives] introducing a comparison of one object with another one of the same kind, as 'Achilles is braver than Ajax', or of one object to more objects of a different kind, as 'Achilles is braver than the Trojans'" (Uhlir: 1883:27).

There is a basic difference between ancient and modern inter-

preters. The authors of the *scholia* to the *Tékhnē* implicitly accept the above definition as a sound one, and limit themselves to explain and enlarge it, making explicit many points which the extremely syncretical form of the definition leaves implicit. From this point of view, the *scholia* are important not only as an integration to Dionysius' brachylogical definition, but also (in a way, mainly) because, together with the text of the *Tékhnē*, they build up the first sketch of what can be called a 'discourse on the comparative' in Western grammatical tradition; this initial discourse can be assembled and reconstructed through the *disiecta membra* offered by the *Scholia*, and integrated with passages by philosophers dealing with a 'metatheory' of the comparative.

Modern interpreters tend to criticize and accuse Dionysius' definition of inadequacy. The editor of the *Tékhnē* for the *Corpus Grammaticorum Graecorum*, Uhlig, judges the definition as "perverse", because "peccasse autem ita technographus mihi videtur, quia ea tantum exempla, quae subigit, in fingendo ὅρῳ respiciebat" (p. LXXXV). Di Benedetto (1958:101), on the other hand, blames the definition for its "inesattezza" and uses it as a further witness of the many faults of the *Tékhnē*, which, in his view, should enable us to deny its authenticity and ascribe it to a later, modest compiler rather than to Dionysius Thrax. As to the last editor, Lallot (1989), author of an outstanding and exhaustive comment to the *Tékhnē*, he views this definition as incomplete and adds that the *Scholia* are ingenious, but are unable to explain the limits of the definition itself.

What all modern interpreters object to in the author of this definition (whether he is Dionysius himself or another compiler is not an important point in our present discourse), is that he is unable, or has chosen not to, base it on the actual linguistic usage of Greek (and, for that matter, of the other languages whose comparative is typically similar to Greek, as all Western languages are). Obviously, the two main points of the definition liable to this criticism are the limitation of comparisons in the one-to-one case and in the homogeneous-objects one: it is difficult to see why comparative sentences such as "Achilles is braver than Ajax and Agamemnon" (with a one-to-two comparison) or "Achilles is braver than Hector" (where the two objects belong to two different kinds, the Greeks and the Trojans) should not be perfectly grammatical and well formed in Greek as well as in any other language; neither is one satisfied, of course, by the explanation of a scholiast (*Schol. Vatic.*, cod. C, Hilgard 1901:225, 14 ff.), who, attributing Dionysius' rule to his "philellenism", leads himself to Lallot's reproach that it is an insufficient and naive explanation.

1.2. As it is fairly evident, no one can deny that the definition does not conform to the actual usage of the language and that, from this point of view, it is clearly inadequate, at least according to our contemporary standards of inadequacy of a grammatical definition, as stemming from the postsaussurean requisite of a linguistic science based on *la langue envisagée en elle-même et pour elle-même*. To this objection a counterobjection can be advanced: did this epistemological and theoretical prerequisite hold for the premodern study of grammar and languages and, especially, for Greek grammarians?

Most of all: from an historiographic (or even better, as I prefer to say, from an 'archaeological and genealogical' - obviously à la Nietzsche-Foucault) point of view, the fact that a grammatical theory or definition does not conform to, nor is based on, actual linguistic usage does not seem to me to be, *in itself*, a strong and compelling reason for denying it any originality and theoretical insight into the object defined. One should rather try to contextualize it, to detect the kind of *discourse*, in which it belongs, and to read it within the cultural background and the "climate of opinion", in which it was formed, developed and put forward, until it became generally accepted and paradigmatic. Unless we can prove that this definition was wrong, inadequate, or inexplicable WITHIN Greek grammatical culture, my impression is that we do not have enough grounds to condemn it.

It is my intention to propose a hypothesis which will show that Dionysius' definition is perfectly coherent and understandable (which does not imply, of course, that we have to accept it as a satisfactory definition of the comparative by our own standards) from the viewpoint of the procedures of Greek grammarians. My hypothesis will determine the origins of this definition as stemming from some easily detectable areas of Greek culture, which were not limited to the field of grammar, but were so important as to influence writers of grammatical treatises, who were not yet bound to the concept of *la langue en elle-même et pour elle-même*.

2. A preliminary remark can be useful. It is a fact that the comparative was brought to the attention of grammarians only at a relatively late period, and with a noticeable delay, compared to other grammatical categories.

In the age of the Sophists, and up to the period of Stoic theorization on grammar, nearly all main grammatical categories had been recognized and defined by philosophers, who were interested in language: Protagoras, the theoretician of correctness of speech

(ὁρθόεπεια), proposed a division of *lógos* into four kinds (or seven, according to another source), but we also know that the division of *lógos* was a widespread object of research (D.L. IX 54 = 80 A 1 D.-K.; cp. also 80 A 25); he divided grammatical gender into the three species: masculine, neuter, and feminine (Arist., *Rh.* 3, 1407 b 6 = 80 A 27), a division whose popularity is witnessed by Aristophanes' parody in the *Clouds*; Diogenes Laertius (IX 52) reports that Protagoras had been the first to divide the parts of time (μέρη τοῦ χρόνου), a statement which is highly probable to be referred to verb tenses, as a division of physical time would not have been viewed as a novelty. Lastly, he is certain to have theorized about modalities, if not about moods, of verb, as is shown from his famous criticism of Homer for employing an imperative referring to a goddess (80 A 29).

The distinction between noun and verb was given the status of a semantic definition in Plato's *Sophist* in the context of a theory about the genesis of *lógos*, before being formalized and made canonical in Aristotle's *de interpretatione*.

As compared to this abundance of grammatical systematizations, a theory of the comparative had to wait until Dionysius Thrax, in whose *Tékhnē* the first definition of this category is to be found. This fact cannot but look peculiar, if we consider that a 'comparative point of view' was one of the most characteristic features of Greek thought, especially, but not only, in its philosophical side, starting, at least, with Anaxagoras, who sees in the activity of συγκρίνειν one of the functions of the νοῦς, together with the opposite one of διακρίνειν and ἀποκρίνειν.

Of course, this delay can be due to mere chance or to lack of explicit evidence: after all the mention of the comparative in the grammatical papyrus *P.Heid. Siegmán 198*, n. 12 Wouters, (whose chronological relation to the *Tékhnē* is difficult to determine, due to the doubts about the latter's attribution to Dionysius) and the *scholia* to the *Tékhnē* witness the working out of a theory of comparative more gradual and widespread than would result from the brachylogical definition of Dionysius. Therefore, we can suppose that Dionysius' definition was not totally original, but was rather the culmination of an earlier process of theoretical meditation.

On the other hand, a whole series of historiographical considerations allows us to suppose that this delay was not totally due to chance and that, quite the contrary, it is the signifier of an initial uncertainty which brought about a postponement of the theoretical

prerequisites necessary for the definitory and functional elaboration of this fundamental category.

2.1. It is not too difficult to assume that grammarians and philosophers were aware of the existence of the comparative as a distinct element of grammar before Dionysius Thrax introduced it into grammatical technical treatises; this can be witnessed, for instance, by Plato's reference, in the *Philebus*, to the two Forms of More and Less (τὸ μᾶλλον καὶ τὸ ἥττον), as resulting from the second ontological principle, the Great-and-Small (τὸ μέγα καὶ μικρόν), which Plato exemplifies with the very use of comparatives ("hotter and colder" and so on: 22 C ff.). Therefore, the *idea* of the comparative has to be traced back at least to Plato's time, but in an implicit way it was already implied in Parmenides' denegation of those parts of ontology where Being should be qualified in terms of More and Less, rather than in those of the Plenitude of Being (οὐδέ τι τῇ μᾶλλον, τό κεν εἴργοι μιν συνέχεσθαι, οὐδέ τι χειρότερον, πᾶν δ' ἐμπλεέον ἐστὶν ὄντος: 28 B 8, 23-24). On the other hand, its *thought* and, as a consequence, its technical definition and theory could only have been possible as the result of its *prob-lematization* (I am obviously referring to Foucault's three well known historiographical categories exposed in *Discourse and Truth*).

Before a theory of the comparative could be shaped in terms of a chapter of technical grammar, a *metatheory* of the comparative had to be worked out in the wider field of philosophy, especially within ontology and categorial logic, as well as in other more technical sectors of Greek theoretical thought, such as mathematics, geometry, physiology, medicine, etc. For reasons of space I cannot deal with this aspect presently and will therefore include it in another essay I am preparing; here I will limit myself to a short allusion to a particular aspect of this wider topic.

2.2. It is important to recall that, when Aristotle systematizes the logical (as well as ontological) category of the *relative* (πρός τι) in the *Categories* (VII, 6 a 36), he gives the comparative as his first example of this category: "the greater" is said to be greater by reference to something outside it. For, indeed, when we call a thing 'greater', we mean by that greater *than* something" (τὸ μείζον τοῦθ' ὅπερ ἐστὶν ἐτέρου λέγεται. τινὸς γὰρ λέγεται μείζον). Therefore the comparative is, in his view, a relative concept; that is to say, a concept which "is said to be such as it is from its being of some other thing or, if not, from its being related to something in some other way" (Cook 1938): for this reason Aristotle is eager to underline that substance does not admit

of "more and less", that is to say of being inflected in the comparative, with some exceptions (8 a 14). This idiosyncratic rule will be inherited by later grammarians, such as, for instance, the author of the *Scholia Marciana* (VN) to Dionysius' *Tékhnē*, who states that comparatives are not derived from proper and common nouns (that is, from signifiers of substance), but only from adjectives: there is no comparative derived from the word "orator" (ρήτορ), but only from its adjective ῥητορικός, because the noun ῥήτορ refers to the "complete orator", τὸν τέλειον ῥήτορα (Hilgard 1901:371, 10 ff.). It is evident that this rule derives from the Aristotelean theory of the relative.

It is well known that the development of the category of relation required a long, gradual elaboration, begun by Plato in the *Sophist* (255 c-d: it has to be distinguished between those beings which are said "in themselves" – αὐτὰ καθ' αὐτά – and those, which are said only "in some relation": πρὸς ἄλλα) and continued inside the Academy, before being made canonical and formalized in the context of the foundation of formal logic in Aristotle's *Organon*.

It is my intention to develop in another essay the connections between the development of an independent category of relation and the "battle of giants", which opposed the socratic-platonic-academic-aristotelean line of thought to sophistic panrelativism, as expressed in Protagoras' well known formulas "πάντ' ἀληθὴ" ("all propositions are true") and "οὐκ ἔστι ἀντιλέγειν" ("it is impossible to contradict"). In the line of attack against the Sophists, it seemed essential to establish the ontological, as well as ethical, view of an objective and absolute Truth (ultimately going back to Parmenides' *Alétheia* as radically opposed to *dóxa* and to merely phenomenal appearances) in opposition to the relativism implied by sophist *logology* (Cassin 1986:17). According to this view, truth is a discursive, (inter)subjective matter, built up inside each single speech act and based upon Protagoras' principle that "what appears to me, it is-to-me"; the ultimate implicit result is that there is no Truth-in-Itself, no αὐτὸ καθ' αὐτό, but only just as many true-for-someone propositions as there are different acts of speech between participants in a dialogue situation.

The relevance of this "battle of giants" (γίγαντομαχία) to the development of a (meta)theory of the comparative is to be seen in Protagoras' radical view: "I cannot conceive that one of these men [i.e. the sick and the healthy, who have different sensations of the same object, e.g. wine] can be or ought to be made wiser than the other...these experiences which the inexperienced call true, I main-

tain to be only better, and not truer than others": *Tht.* 166 E ff., Jowett 1964). To this questioning of the absolute character of truth, upon which a comparative view of reality ultimately rests, Aristotle felt compelled to reply in an equally radical and definitive way that "there is More and Less in nature" (τό γε μᾶλλον καὶ ἧττον ἔνεστιν ἐν τῇ φύσει: 1008 b 32): notice that Aristotle introduces this statement in order to confute the denegation (implied in Protagorean relativism) of the principle of non contradiction; in this context Aristotle also lays down the foundations of a referential semantics as opposed to the Sophists' view of language as a *plaisir de parler* (Cassin 1986; Cassin and Nancy 1989).

As a result, the only way to defeat sophistic relativism was to accept the existence of relative as a concept and as an aspect of reality, but, at the same time, to demonstrate that relation is only *one* aspect of reality, and that, beside it, the vast and fundamental field of the In-itself, αὐτὸ καθ' αὐτό, extends to grant an objective ground for the establishment of absolute Truth as well as of absolute Good. It was important to detect the relative as an independent, but limited and formally well defined, category, identified, and therefore isolated, within precise borders, πέρατα, which allow it to be distinguished from the category of the Absolute, in order to avoid any lapse into the dangerous indefiniteness (ἄπειρον) of Protagorean πάντ' ἀληθὴ or Anaxagorean πᾶν ἐν παντί.

The ontologically priority of the In-itself, be it the world of Platonic Forms or Aristotelean *ousíai*, does not allow of more and less: no Form can be more or less than what it is, neither can a substance ("man") be more or less: there is no such thing as a 'more or less man': the More and the Less only apply to qualities ("warmer and colder", "quicker and slower") and quantities ("double and half") and are relative concepts: something can only be warmer in relation to something colder; "double" can only be such as opposed to "half", "right" as opposed to "left".

From this it is easy to infer that a grammatical theory of comparative could only have been established after a *metatheory* of comparative, based on the logical category of relation, had been completely developed. If the cultural battle had been won by Protagoras and the Sophists, not to mention other minor schools, which were defeated by the Platonist and/or the Aristotelean lines, it can be presumed that grammarians would have defined the comparative in an utterly different way.

3. After this digression we can go back to the problem of interpreting the far from easily understandable definition of Dionysius.

The first clue that it was coherent and justifiable inside Greek culture, comes from the fact (which has not been so far recognized) that it is confirmed by analogous definitions in the parallel field of treatises of rhetoric, where the equivalent of grammatical comparison, σύγκρισις, was accurately dealt with and defined.

"Comparison" (σύγκρισις) was a rhetorical figure mainly used in the literary genre known as *encomion*, where two objects are compared in order to establish the excellence of either of the two: Theon (Spengel 1853: II, 112 ff.) defines it as a discourse establishing the better and the worse (τὸ βέλτιον ἢ τὸ χεῖρον παριστάς). Aphthonius' definition (ib.: 42) is particularly significant from our viewpoint: σύγκρισις is a "comparative discourse, which by juxtaposition concludes attributing the more or the equal to the object undergoing comparison" (λόγος ἀντρεξαστικός ἐκ παραθέσεως συνάγων τῷ παραβαλλομένῳ τὸ μείζον ἢ τὸ ἴσον): the translation omits to stress the many indications to the technical use of nearly each lexical item in sciences other than rhetoric: the verb παραβάλλειν, for instance, was the technical term referring to the well known Pythagorean-Eudoxean-Euclidean method of "applying" a figure on a line or a figure on another figure (itself a method showing several affinities with a theory of comparatives, as I intend to show elsewhere); as to ἀντρεξαστικός, it was an established technical term for the grammatical discourse about comparative, as results from its use in the *scholia* and commentaries to Dionysius, such as, e.g., the *Commentar. Byz.* (Hilgard 1901:573, 25 ff.), whose author practically quotes the rhetorical definition: "comparison is a comparative (ἀντρεξαστικός) discourse, which by the juxtaposition of another person, concludes attributing the 'more' to the object undergoing comparison"; the noun, from which it derives, ἀντρεξάσας, signifies "discrimination (of a part of speech)" in Apollonius Dyscolus (Uhlir 1910:220, 4), but it was also a technical term of arithmetics, where it referred to the comparison of an integer with the sum of the preceding integers (cp., e.g., *Theol. Ar.* 10).

What is closely connected to our analysis, is that the rhetoricians are very careful to emphasise that σύγκρισις, as a rhetorical figure, has to be introduced between *two*, and only two, objects: as Hermogenes states in his *Progymnasmata* (ib.: 14), "we compare a town to a town, ... and a kind to a kind, or a food to a food, ..., justice and wealth, etc.": that is to say, the comparison is ἐνὸς πρὸς ἓν, as Theon: 114, puts it, with the phrasing, that Dionysius uses in his

grammatical definition. According to Aphthonius (*loc. cit.*) the comparison is between beautiful and useful objects, or between base ones, also between useful and harmful, smaller and bigger objects. From these examples it also appears essential, even if it is not explicitly stated, that the comparison has to be performed between objects belonging to the same species, as two different instantiations of the species ("town to town") or as contraries and contradictories ("useful to harmful"). I wish to quote an example, which will be useful to our analysis of Dionysius' definition. Aphthonius (p. 43) deals with the case of a comparison between two objects, that are not, strictly speaking, homogeneous. The comparison between Achilles and Hector from the point of view of their virtue (ἀρετὴν ἀρετῇ συγκρίναι), seems to be impossible according to the rules, due to the fact that the two men were from different countries, a criterion, which reminds of that implied by Dionysius and the *scholia* that the comparative only applies between two Greeks (Achilles and Ajax), but not between a Greek and a Trojan (Achilles and Hector); in spite of this, we can still compare the two men, according to Aphthonius, because this fundamental difference (an element of heterogeneity) is compensated by a whole set of similarities (homogeneity): they were both born in renowned cities, they both descend from Zeus, they were both brought up as brave men and, therefore, having received the *same* training as far as courage is concerned, they achieved the *same* glory; they showed the *same* strength in the *same* war; therefore, as they had a similar life and a similar death, they are *similar* (ὁμοίου). We should note the recurrent use of the words "same" and "similar", two concepts which were central in the theory of comparative, as we will see; at any rate, from this argument, it appears that the criterion of homogeneity was an essential part of the rhetorical theory of comparison, as it was in the grammatical one. Another indirect proof comes from an interesting passage of Theon's *Progymnasmata* (p. 114), where he sets up the rules to follow in the case of a comparison between two sets of objects belonging to two different species, as when one would compare the two species of men and women, in order to determine which one is the more courageous. In this case, it is necessary to choose, within each of the two species, the ἀκρότατα, that is the two persons most endowed with the quality under discussion, in this case courage. A comparison between the most courageous man and the most courageous woman will extend this quality to the whole set: if Themistocles, the most courageous man, proves to be more courageous than Artemisia, the most courageous woman, it is possible to infer that the whole species of men excels in courage over the

whole species of women. Another criterion is quantitative: the species, that contains a larger number of people showing the quality exceeds the other: e.g., even if Tamaris is found to be more courageous than Cyrus, this does not necessarily imply that the species of women is more courageous than that of men, since it would only lead to the conclusion that one or two women are courageous, whereas the number of courageous men is much larger.

3.1. The obvious conclusion is that the two definitions of σύνκρισις, the rhetorical and the grammatical, match and confirm each other as acceptable and coherent definitions within the conceptual background of Greek culture.

Two hypotheses are possible as a consequence of what we have just seen. On both a philosophical and a chronological basis, we might suppose either that the grammatical definition of comparison influenced the rhetorical one or vice versa, that is that the latter was derived from the former. It might also be presumed, of course, that both definitions were formed together within the circles of grammarians and rhetoricians, due to the strong interconnections between the two disciplines.

At a deeper level, though, I think that I can advance another hypothesis, which seems to me to be highly probable and convincing, as it would allow us to trace *both* definitions to a more general conceptual frame, which was derived from one of the leading sectors of Greek thought in the period during which we may presume that a definition, or a set of tentative definitions later abandoned in favour of the Dionysian one, were in the process of being formed. I am referring, strange as it can seem, to the field of mathematics and, especially, *geometry*.

3.2. We need to try to abandon reading Dionysius' definition with our modern, postsaussurean eyes and give ourselves the difficult task of going through it with the textual memory of a Greek reader, who was (or was expected to be) an educated person, whose *paideia* had been shaped around the disciplines and the conceptual frame required, e.g., by Plato's *Republic* and *Laws*, with philosophy and mathematical sciences forming the core of his intellectual curriculum: "No one will be allowed into the Academy, unless he is knowledgeable about geometry", as Plato wanted to be inscribed on the threshold of his school, thus hinting at the basic requirement of his didactic program.

Two important concepts derived from Foucault (1969) are rel-

evant to an "archaeology" (not simply a history) of the discourse about comparatives.

The first has to do with the supposed unity of a text, which at closer inspection will appear as a relative and variable one, since every book "est pris dans un système de renvois à d'autres livres, d'autres textes, d'autres phrases: nœud dans un réseau"; its relative unity "ne se construit qu'à partir d'un champ complexe de discours" (Foucault 1969:34).

From this point of view, Dionysius' definition will appear as a hypertext transposing into a grammatical context a series of hypotexts derived from other disciplines, which he considered particularly appropriate to form the conceptual frame for a rational definition of the "comparative noun", συγκριτικὸν ὄνομα. The lexical items involved in the definition (and we know how important the wording of definitions was in a postaristotelean definitory procedure) acquire, in this perspective, a double function. On a merely semantic level, their meanings contribute to the general meaning of the definition; on a deeper, textual and *discursive* level, they become signifiers of the conceptual field, from which the definition is derived: in other words, they function as a sort of quotation of definitions of objects pertaining to other sciences, taken as warrants to the rationality, scientific reliability and soundness of the definition itself.

It is to be underlined, again following Foucault (1969:39), that the analysis of a discourse (in our case the discourse on comparatives) does not only imply an understanding of what has been said, but also, maybe even mainly, of what has *not* been said: "comment se fait-il que tel énoncé soit apparu et nul autre à sa place?". Dionysius' definition of the comparative might have been another one, derived from another conceptual frame (or other conceptual frames) and supported by principles of other disciplinary fields: this does not mean choosing a counterfactual historiographical perspective, but the more proficient procedure by which to understand, indirectly rather than counterfactually, why *that* particular conceptual frame was chosen, rather than (an)other one(s).

4. The ideal Greek reader of the *Tékhnē*, highly educated and geometrically trained, or at least familiar with mathematical notions, whose textual memory we are trying to reconstruct, certainly recognized the striking similarity (nearly a word for word quotation) between the wording of Dionysius' definition of the comparative and the Eudoxean definition of "ratio" (*lógos*), as we know it from Euclid's *Elements*, V def. 3:

Λόγος ἐστὶ δύο μεγεθῶν ὁμογενῶν ἢ κατὰ πηλικότητα ποῖα σχέσις.

"A ratio is a sort of relation in respect of size between *two* magnitudes of *the same kind*" (Heath 1926:I, 114: my italics).

This definition had become canonical and can be found in practically all mathematical textbooks, such as Theon Smyrnaeus ("Ratio is a sort of relation of two analogical homogeneous terms between each other": Hiller:73, 16), Nicomachus of Gerasa ("Ratio is a relation of two terms between each other": *Ar. I, XXI*) and Iamblichus (*in Nic.* p. 98, 15 ff.). The definition given by Iamblichus is of particular interest, because it provides a series of clear examples:

"The proportional ratio of two homogeneous terms is a reciprocal relation between them. 'Homogeneous' is added, because it is befitting to compare (συγκρίνειν) objects falling under the same kind, such as, e. g., a mina to a talent, whose common kind is weight, a line to a surface or a solid, because their common kind is magnitude.... As to non homogeneous objects it is impossible to know how they relate to one another, as in the case of a cubit to a *kotule* [a dry measure]..."

4.1. A mathematical ratio, as defined by Eudoxus, in order to be recognized as such, must satisfy two conditions. The first is that it has to be established between *two* and *only two* magnitudes: if more than two magnitudes are introduced, their relationship is no more a ratio, but a (possible) proportion, i.e. a "same-ratio" relationship:

"A proportion in three terms is the least possible." (*Elem.* V def. 8)

This rule (or, rather, definition) is set up by Euclid after defining proportion:

"Let magnitudes which have the same ratio be called proportional."

(Τὰ δὲ τὸν αὐτὸν ἔχοντα λόγον μεγέθη ἀνάλογον καλεῖσθαι: V def. 6; sameness of ratio is dealt with in def. 5, which I will quote later).

After stating this numerical rule in order to distinguish between ratios and proportions, Euclid proceeds to give a more precise definition of three- and four-term proportions:

"When three magnitudes are proportional, the first is said to have to the third the duplicate ratio of that which it has to the second" (V def. 9)

and:

"When four magnitudes are <continuously> proportional, the first is said to have to the fourth the triplicate ratio of that which it has to the second, and so on continually, whatever be the proportion" (V def. 10).

Euclid is very careful to observe this sort of numerical distinctions in his propositions. For instance, when he proves the rule which allows to find the greatest common measure for numbers in book VII and for magnitudes in book X, he first proves it for two objects:

"Given two numbers not prime to one another, to find their greatest common measure" (VII prop. 2; cp. X prop. 3 for magnitudes)

and then he extends the proof to the case of three numbers:

"Given three numbers not prime to one another, to find their greatest common measure" (VII prop. 3; cp. X prop. 4 for magnitudes)

which, as Heron had already noticed (quoted by Heath 1926:II, 302), can be used to find the greatest common measure of as many numbers as we please, because any number measuring two numbers also measures their greatest common measure; and hence we can find the greatest common measure of pairs, then the greatest common measure of pairs of these, and so on, until only two numbers are left and we find the greatest common measure of these. As Heath notes, Euclid tacitly assumes this extension in VII prop. 33 ("Given as many numbers as we please, to find the least of those which have the same ratio with them"), where he takes the greatest common measure of as many numbers as we please.

4.2. The second, and even more important, condition for the establishment of a ratio (as well as, of course, of a proportion) requires the terms to be *homogeneous*. This is one of the reasons why Euclid supplements his definition of ratio with def. 4, stating that:

Λόγον ἔχειν πρὸς ἄλληλα μεγέθη λέγεται, ἃ δύνανται πολλαπλασιαζόμενα ἀλλήλων ὑπερέχειν.

"Magnitudes are said to have a ratio to one another which are capable, when multiplied, of exceeding one another".

to which the famous "eudoxean-archimedean postulate" follows:

"Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order" (V def. 5).

4.3 A particularly clear example of the care taken by Euclid to observe this condition in his demonstrations, but also of the difficulties caused by this condition, can be seen in the two procedures followed in his proves of X prop. 5, and X prop. 9.

The well known prop. 5 states that:

"Commensurable magnitudes have to one another the ratio which a number has to a number."

The historical and cultural importance of this theorem lies in the fact that it gives a rational, geometro-algebraic, demonstration to one of the basic principles of Pythagorean as well as Platonic ontological view (shared by the great majority of Greek schools of thought) that reality is expressible in numerical terms, as number to number.

In modern terms, the argument of prop. 5 runs as follows (I am quoting from Heath 1926:II, 25).

If a , b be commensurable magnitudes, they have some common measure c , and

$$\begin{aligned} a &= mc, \\ b &= nc, \end{aligned}$$

where m , n are integers.

It follows that

or, inversely

and also that

so that, *ex aequali*

$$c : a = 1 : m \dots \dots \dots (1)$$

$$a : c = m : 1,$$

$$c : b = 1 : n,$$

$$a : b = m : n.$$

As Heath observes in his commentary, in stating the proportion (1), Euclid is merely expressing the fact that a is the same multiple of c that m is of 1, resting the statement on the definition of proportion in VII def. 20, which is something like a repetition for the particular field of *numbers* of the general definition of proportion given in V def. 5 for *magnitudes*. It is a well known fact that Euclid defines propor-

tions *twice*: in book V for magnitudes and in book VII for numbers; this apparently strange procedure might be due to the fact that most mathematicians and philosophers (Aristotle was the strongest supporter of this view) were careful to distinguish *numbers* from *magnitudes*, on the basis of their view that magnitudes can be incommensurable, whereas numbers do not know the embarrassing phenomenon (a logical as well as ontological scandal) of incommensurability. Therefore, as Heath continues, in the demonstration of prop. 5 the statement of the proportion is not legitimate (as c , a are not numbers, but magnitudes) unless it is proved that it is true in the sense of V def. 5 with regard to magnitudes in general, the numbers 1, m being *magnitudes*. Again in Heath's view, Euclid ought to have proved that magnitudes which are proportional in the sense of VII def. 20 are also proportional in the sense of V def. 5, or that the proportion of numbers is included in the proportion of magnitudes as a particular case.

Euclid must have been conscious of these difficulties, as can be seen by his procedure in the proof of the other theorem (X prop. 9), stating that:

"The squares on straight lines commensurable in length have to one another the ratio which a square number has to a square number; and squares which have to one another the ratio which a square number has to a square number will also have their sides commensurable in length[...]"

This theorem is a logical consequence of the previous one, as it results from its application at the beginning of the demonstration. In modern symbolism, the proof runs as follows:

If a , b be straight lines, and, as a consequence of X prop. 5,:

$$a : b = m : n,$$

where m , n are numbers,

then

and conversely.

$$a^2 : b^2 = m^2 : n^2,$$

But this inference was by no means so easy for Euclid owing to the fact that a , b are straight lines, and m , n numbers. He has to pass from $a : b$ to $a^2 : b^2$ by means of VI prop. 20, *por.* ("it can be proved in the case of quadrilaterals that they are in the duplicate ratio of the corresponding sides"): the square on a is to the square on b in the duplicate ratio of the corresponding sides a , b . On the other hand, m , n being *numbers*, it is VIII prop. 11 ("Between two square numbers there is one mean proportional number, and the square has to the

square the ratio duplicate of that which the side has to the side") which has to be used to show that $m^2 : n^2$ is the ratio duplicate of $m : n$. Then, in order to establish his result, Euclid assumes that, if two ratios are equal, the ratios which are their duplicates are also equal, an assumption which is nowhere proved in the *Elements*, but can be an easy inference from V prop. 22 ("If there be any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will also be in the same ratio *ex aequali*"): see Heath (1926:III, 31 and II, 242 ff.).

From the point of view of the homogeneity-condition, it is important that Euclid offers two different proves, one for magnitudes, and another for numbers.

4.4. There are many instances of this procedure, by which Euclid proves that two different figures are to one another as one of their components, either their bases or their volumes and areas, proving the ratios separately for each set of homogeneous objects. A simple example will suffice.

In VI prop. 1 Euclid proves the theorem:

"Triangles and parallelograms which are under the same height are to one another as their bases."

Given two triangles (ABC , ACD) and two parallelograms (EC , CF), constructed on the same height, Euclid begins his demonstration by proving that the two triangles are equal, providing the conditions stated in I prop. 38 through the production of the two bases in both directions; successively he refers to well known V prop. 5 in order to prove that whatever multiple the base HC (obtained through the production of the original base towards the left), that multiple also is the triangle ALC (constructed on the new, produced base, of the original triangle ACD), that is to say:

If the (new) base HC is equal to the base CL (i.e. the new base towards the right), then the triangle AHC (on the base HC) is also equal to the triangle ACL on the base CD ;
If the base HC is in excess of the base CL , then the triangle AHC is also in excess of the triangle ACL ;
And if less, less.

According to V def. 5, the ratio between the equimultiples of the two bases and the two triangles allows to prove that, as the base BC

is to the base CD , so is the triangle ABC to the triangle ACD . At this point, Euclid proves that, as the parallelogram EC is double of the triangle ABC and the parallelogram FC is double of the triangle ACD (the proof has been given in I prop. 41), and as parts have the same ratio as the same multiples of them (proven in V prop. 15), it is possible to infer that:

As the triangle ABC is to the triangle ACD , so is the parallelogram EC to the parallelogram FC .

And: having been proved during the demonstrative procedure that:

As the base BC is to the base CD , so is the triangle ABC to the triangle ACD

and that:

As the triangle ABC is to the triangle ACD , so is the parallelogram EC to the parallelogram FC .

Therefore, also:

As the base BC is to the base CD , so is the parallelogram EC to the parallelogram FC .

It can be seen that the demonstration proceeds proving successively and separately that there is a ratio between base and base, triangle and triangle, parallelogram and parallelogram, thus leading to the result that the same proportional ratio obtains between the two figures and between them and their bases. We should also notice the accuracy of the wording, which stresses the field of pertinence of the three different ratios through the repetition of the words relating to the three sets of two, homogeneous objects: the *triangle...the triangle; the parallelogram... the parallelogram; the base... the base*. This wording, careful to the point of risking to appear monotonous to a lay reader, witnesses the careful distinctions which Euclid presumes to be necessary in order to meet the condition of homogeneity in the establishment of ratios and proportions.

4.5. From our point of view, it is relevant to observe that the proof of a proportional ratio occurring between two or more sets of objects,

makes use of the three expressions "is/are equal to", "is (are) in excess", "is less than". Such procedure will be repeated several times in the propositions based on the method of exhaustion of book XII, where the proportions between figures, volumes, and areas are proved through a *reductio ad absurdum* of the opposite hypothesis: in order to prove that the area or volume *A* of a given figure is equal to an area and volume *B*, it is proved that it is *not* the case that $A > B$, showing that, whatever difference obtains between *A* and *B*, it will necessarily be found to be less than any arbitrarily chosen magnitude. This is an application of the "antanairetic" process proved in the fundamental theorem X prop. 1, according to which, at the end of an in(de)finite process of subtractions between two unequal magnitudes, there will be left some magnitude which will be less than any lesser magnitude set out.

The importance of this proposition was outstanding, because it allowed Eudoxus, and later Euclid and Archimedes, to go back to the Zenonian-Anaxagorean infinitesimal perspective after its challenge by the Atomists, who wanted to arrest the process of infinite division to an atomic limit. As Anaxagoras had put it, "bigness and smallness are unlimited" (59 B 1) and "within the Small there is no smallest, but only the infinitely smaller; the same holds for the Great: from this point of view the Small is equal to the Great and any object is both great and small" (τοῦ μικροῦ οὐκ ἔστι τό γε ἐλάχιστον, ἀλλ' ἑλασσον δέ... ἀλλὰ καὶ μεγαλύτερον αἰεὶ ἔστι μείζον. καὶ ἴσον ἐστὶ τῷ μικρῷ πληθύνος, πρὸς ἑαυτὸ δὲ ἑκάστὸν ἔστι καὶ μέγα καὶ μικρόν: B 3), which meant that there is no limit to infinite division of magnitudes: the way was open to the geometrical analysis of the infinite and the infinitesimal, which will be brought to perfection through the method of exhaustion by Eudoxus, Euclid, and Archimedes. On the other hand, the typical refusal of Greek thought to admit of the infinite (the *Apeiron*, which opened the negative column of the Pythagorean table of opposites), as exemplified by Aristotle's admission of the infinite as only in potency, but never in act, did not favour the full development of an infinitesimal calculus. As historians of Greek mathematics have pointed out, the lack of the modern concept of "limit" (in the process of exhaustion, the circle, rather than being actually exhausted by the regular inscribed polygons with successively smaller sides, functions as their limit) was the cause of the adoption of a negative proof *ad absurdum* or *impossible*, as well as of the necessity to prove each theorem based on the method of exhaustion separately, rather than building a universal theorem covering all possible cases. As Enriques and Santillana (1936:51) have pointed out, the principle of contradiction

and the *reductio ad absurdum/impossible*, these two formidable tools of Western logic, have been brought to a formal rigour (through Zeno's arguments) as a result of the discovery of the infinitesimal and related problems: each time it is required to prove an equality (of surfaces, volumes etc.) with recourse to the infinite, it is possible to avoid "la démarche aventureuse" of summing up an infinite number of magnitudes and of bisecting a whole into an infinite number of parts by proving, with a finite number of logical passages, that *it is not the case* that the magnitudes, whose equality is presumed, are unequal, because neither is bigger than the other (on this extremely important point, cp. *inter alios*: Zeuthen 1896:64 ff.; Frank 1923:46 ff.; Tannery 1930 chapter X; Rufini 1961; Mondolfo 1935:89-145; Id. 1965:196 ff.).

The three sets of expressions employed in Euclid's exhaustion propositions, as well as the problems connected with their use, are naturally at the core of the field of the (grammatical) comparative, as well as of a metatheory of comparatives, thus showing how strong the links were between the geometrical theory of ratios and proportions and the theory of comparison: we have seen before that Iamblichus referred to "comparison" (συγκρίσις) in his definition of *lógos*.

5. There had been anticipations of this procedure in the Pythagorean tradition.

Alexander of Aphrodisia (in *Metaph.*:38, 10 ss.), after reporting the main features of Pythagorean numerology and theory of musical harmony, gives us a sketch of their theory of a heavenly harmony. Its presupposition is that heavenly bodies move around the centre $\epsilon\nu \acute{\alpha}\rho\mu\omicron\nu\iota\alpha$ and that some of them rotate quicker ($\theta\acute{\alpha}\rho\tau\omicron\nu$) than others, whose rotation is slower ($\beta\rho\alpha\delta\acute{\upsilon}\tau\epsilon\rho\omicron\nu$); their movements produce a sound, which is low in the case of the slowest bodies, high in that of the quickest ones. These sounds are produced according to a ratio among distances so as to cause an $\epsilon\nu\alpha\rho\mu\omicron\nu\iota\omicron\nu \acute{\eta}\chi\omicron\nu$. For instance: given that the distance of the sun from the earth is double that of the moon, triple that of Venus, quadruple that of Mercury, and so on according to an "arithmetical ratio", the most distant bodies have a quicker rotation, whereas the nearer they are the slower they move.

It is easy to see that this theory applies three parameters: distance, speed, and the nature of the sound, according to a ratio of direct proportion; the greater the distance, the greater the speed, the greater the height of the sound. That is: + distance : + speed : + height = - distance : - speed : - height.

For example: the sun's distance from the earth is double that of

the moon; therefore its speed is double the moon's and its sound is twice as high. It is also to be observed that the scale of distances proceeds according to intervals which are multiples of unity, up to 4: 1, 2, 3, 4 = 10, the famous Pythagorean decade, the perfect number. Moreover, the ratios 1:2, 2:3, 3:4 had been established as measures of the three major musical intervals.

As to the existence of a ratio between speed and the nature of sound, it had already been stated in Pythagorean musical theories, e.g. by Archytas, according to whom the sounds produced by quick and strong hits are heard as high, whereas sounds produced by slow and weak hits are heard as low: B 1.

5.1. The homogeneity-condition soon became one of the most deeply established principles of mathematical sciences, contributing to generalize and impose the Eudoxean-Euclidean theory and definition of ratio and proportion, which replaced a precedent definition (possibly attributable to Theaetetus), based on *antanaíresis*, according to which "magnitudes have a proportional ratio between one another, when they have the same division (*anthuphaíresis*)" (Alex. Aphr., in *Top.*:545, 12 ff.); this pre-Eudoxean definition is still present in Aristotle (*Top.*:158b 29 ff.): see Becker 1933.

The homogeneity-condition, as stated explicitly in Euclid's V def. 3 and further expanded in the so called Archimedean axiom of V def. 4 and 5, was introduced mainly with the aim of coping with the new requirements and problems induced by the discovery of irrationals and incommensurables, as well as by that of the infinite and infinitesimal, following Zeno's arguments and Anaxagoras' theory of the "infinitely greater and infinitely smaller": this condition "emphasises the fact that the term *ratio*, as defined in the preceding definition, includes the relation between any two *incommensurables* as well as between any two commensurable finite magnitudes" and, at the same time, it excludes "the relation of a finite magnitude to a magnitude of the same kind which is either infinitely great or infinitely small" (Heath 1926:II, 120).

It thus became the most important *diorismós* of "Euclidean" geometry. As is well known, Euclid's method is characterized by the use of *diorismoi*, that is idiosyncratic restrictions, which determine the *possibility* of the construction or of the demonstration (see, *i.a.*, Zeuthen 1910:403 ff.; Heath 1926:I, 129 ff. and *passim*). For instance, the fundamental definitions of *antanaíresis* for numbers (VII prop. 1) and for magnitudes (X prop. 2) prescribe the two numbers and magnitudes to be *unequal*; similarly, the demonstration of the rule which

finds the greatest common measure between two numbers (VII prop. 2) or magnitudes (X prop. 3) prescribes the two numbers *not* to be *prime* to one another and the two magnitudes to be *commensurable*. One of the many examples of the use of *diorismoi* can be seen in VI prop. 9: "From a given straight line to cut off a prescribed part", to which the condition is added: "Let the third part be that prescribed".

The importance of *diorismoi* is stressed by Proclus (in *Euc.*:200 ff.), who defines it as that part of the demonstration which declares whether the object of research (τὸ ζητούμενον) is or is not a possible one. Translating Euclid's procedure into the language of Hellenistic, especially Stoic, propositional logic, Proclus states that the *diorismós* "clearly states (δισσαφεί) the possibility of the construction; he uses a verb which in Stoic logic referred to the cogent establishment of the truth of a proposition, as in the "proposition stating the more (ἀξιοῦσα δισσαφείν τὸ μᾶλλον)", mainly directed against the Sceptics' denegation of a criterion of truth (see Berrettoni 1997) and he further phrases the *diorismós* as a conditional (συντημένον); e.g., Eucl. I prop. 1 ("On a given finite straight line to construct an equilateral triangle") can be read, according to Proclus, as: "if it is a finite straight line, then it is possible etc.", because, if it is not straight, it is impossible to apply a triangle to it (as triangles are contained by rectilinear angles) and, if it is not finite, the construction is still impossible, as it is impossible to construct an angle save on a point, and the point is not the limit of an unlimited line.

6. Both the homogeneity-condition and the necessary introduction of *diorismoi* into the demonstration of scientific propositions were not limited to the technical field of geometry, but became a general epistemological principle of all Greek theoretical sciences. A single example will suffice here.

In a famous passage of the *Laws* (819 E, ff.), Plato, dealing with the phenomenon of (in)commensurability, states the requisite, obviously derived from geometers and mathematicians, that "length is naturally commensurable with length, and breadth with breadth, and depth in like manner with depth". He then refers to a widespread prejudice and mistake that was due to ignorance of the recent developments of geometrical and mathematical research (such as Theaetetus' research on irrationality inside the Academy):

"some things are commensurable and others are not...; concerning length and breadth when compared with depth, or breadth and length when compared with one another, are not all the Hellenes

agreed that these are commensurable with one another in some way?... But if they are absolutely incommensurable, and yet all of us regard them as commensurable, have we not reason to be ashamed of our compatriots... is not this one of the things of which we were saying that not to know them is disgraceful...?" (820 A ff., transl. Jowett 1964)

In this passage two fundamental points are alluded to: the first is the prejudice (probably stemming from a misunderstanding and oversimplification of the Pythagorean principle: "all things are numbers") that all magnitudes are commensurable.

The second mistake, more important from our present point of view, refers to the practice of measuring, comparing, and inserting into a ratio two *heterogeneous* objects, such as a line or a surface ("length and breadth") with a body ("depth"). Of course, no one would have directly compared a surface with a body, saying, for instance, that a particular square is bigger than a particular cube and so on. But it has been Eva Sachs' merit to find and quote, in her comment to this passage, more sophisticated examples of such practice.

Thucydides (VI 1,2) infers Sicily's size from the period of time necessary to circumnavigate it; this procedure can be viewed as sound, at least empirically, but from a purely mathematical point of view it is mistaken, as it establishes a ratio between the content of a body and its contour; the mistake was openly detected and blamed by geometers, according to Polybius' criticism of the naive attitude of most people, who would tell the area of a magnitude from its *perimeters*, and Quintilianus (I 10, 40) states that the historians' habit (the reference to Thucydides is clear) of inferring *magnitudinem insularum... navigationis ambitu* leads to false conclusions blamed by geometers.

As we have seen above, this mistake was not made by the Pythagoreans dealing with the harmonic laws of the movements of heavenly bodies, who were careful to follow a 'Euclidean' procedure, which establishes proportional ratios between homogeneous objects, thus stating that + distance : + speed : + height = - distance : - speed : - height, only after comparing distance with distance, speed with speed, sound with sound.

6.1. The homogeneity-condition played a fundamental role in the debates concerning the solutions proposed for the problem of squaring the circle, whose importance for the development of the exhaustion-method and of the infinitesimal calculus has often been stressed

by historians of Greek mathematics, among whom Becker's comments prevail (Becker 1933).

As Becker stresses, the tentative solutions proposed by Antiphon, Bryson, and Hippocrates, far from being sophisms (as they have been traditionally considered), paved the way for an infinitesimal point of view, which anticipates Dedekind's principle of continuity, as is particularly clear in the discussions they generated, as reported by Aristotle's commentators. The main object of dispute regarded the distinction between *potential construction* and *actual existence* of the object investigated (a problem which is at the core of Proclus' distinction between *problemata* and *theoremata*: cp. in *Euc.*:77, 7 ff.). The problem is clearly stated by Philoponus (in *APo.*:111, 21 ff.), following his teacher Ammonius, who distinguishes between the "mathematisch abstrakte Existenz" and the "konstruktive Erzeugung" (Becker 1933:376) of a regular polygon whose area is to be made equal to that of a circle: for further details I refer to Becker's analysis.

From our particular point of view, the main aspect of the discussion relates to the declaration that the problem of squaring the circle is an "impossible" one, due to the *heterogeneity* of the geometrical objects involved. Thus Philoponus first uses some Euclidean propositions in order to demonstrate the inadequacy of Bryson's axiom, upon which he had based his solution ("objects which are greater and smaller than the same object are equal"), showing that it is too common (*koinón*) a premise to be the basis of a geometrical proof, and does not apply without qualification; he then adds the definitive objection that the problem is impossible "due to the fact that the two magnitudes, the rectilinear polygon and the circle, are heterogeneous": διὰ τὸ ἀνομοιογενῆ εἶναι τὰ μεγέθη... λέγω εὐθύγραμμον τῷ κύκλῳ. On this point there was practically unanimous agreement within official geometric circles.

An anonymous commentator (quoted by Wallies in the apparatus to his Philoponus' critical edition) adds another criticism: as by definition both the inscribed and the circumscribed polygons touch the circle's circumference at one point for each side of the polygon, Bryson should have stated in which of the intermediate points the intermediate polygon (supposed to be equal to the circle) touches the circle: thus Bryson, failing to state κατὰ ποῖον σημείον the square, supposed to equal the circle's surface, falls on the circle, does not apply methods proper to geometry: in other words, Bryson's proof fails to specify the necessary *diorismós*, thus introducing an indetermined, *adióriston*, and, therefore, impossible problem (in Proclus' wording).

7. I assume that Dionysius' definition of the comparative was shaped around the geometrical, Euclidean definition of ratios, which thus offered the conceptual frame and the ground for a metatheory of comparison necessary for the formulation of a grammatical theory of comparatives.

The two idiosyncratic rules ("ένος πρὸς ἕνα" and "ὁμοιογενή") transpose into a grammatical discourse the rule of geometric and scientific discourses that a *diorismós* is required for the soundness of a theoretical proposition: it is generally held that Greek (and, generally speaking, 'prescientific') grammar had a normative and prescriptive character, frequently at the expense of a consideration of actual usage. I do not intend to deny this traditional view, but I think that speaking of a 'diorismós-oriented' definition is more correct and insightful from an historiographical point of view, as this refers to Greek epistemological categories themselves rather than to ours.

To state this point in Proclus' terms, we can say that the two restrictions were meant as: "If two objects are homogeneous, *then* it is possible to compare them with one another in terms of more and less", or, also: "Two, and only two, objects can be compared in terms of more and less, if, and only if, they are homogeneous".

A further point must be made.

What sort of homogeneity was Dionysius thinking of? The obvious answer, derivable from the example "Achilles is braver than Ajax" and supported by some *scholia*, is that it was considered possible only to compare Greek with Greek, Trojan with Trojan and so on; this interpretation is not to be ruled out, as it is also present in rhetorical definitions of "comparison", but on the other hand it really is too simplistic.

A better direction is hinted at by a *scholion*, and can be integrated with a very clear passage from Plato.

7.1. The *scholion* (Hilgard 1901:371, 29 ff.) sets out the rule that a comparison between two objects can only be introduced "when the quality upon which the comparison is based is presupposed [as previously inherent in the two objects]": προϋποκειμένου τοῦ ἐν συγκρίσει πράγματος, for instance, "Achilles is braver than Ajax" is a well formed comparative sentence, because braveness is presupposed in both Achilles and Ajax; "Odysseus is wiser than Nestor" is equally well formed, because wisdom is presupposed as belonging to both Odysseus and Nestor (προϋποκειμένης τῆς σοφίας). It would be wrong to say, e.g., that Socrates is wiser than Coroebus, since the latter is

demented and, as a consequence, a common basis for comparison would be absent.

In a well known passage of Plato's *Phaedo* (96 A, ff.), Socrates is introduced to synthesise his intellectual biography, starting from the initial period, when he followed the physiological tradition and "had a prodigious desire to know that department of philosophy which is called the investigation of nature". In this period (which Socrates will soon define as a naive one)

"I thought that I understood the meaning of *greater* and *less* pretty well; and when I saw a big man standing by a little one, I fancied that one was taller than the other just by the *head* (κεφαλή), and similarly with horses: and still more clearly did I seem to perceive that ten is more than eight because it has two additional units, and that two cubits are more than one because it is larger by a half of itself" (96 D-E, transl. Jowett 1964: my italics).

Later, when he read Anaxagoras, under the influence of his theory of *noûs* he began reasoning in terms of a rational and teleological rather than material and merely natural causality, but, finding even Anaxagoras' theory too spoiled by a naturalistic explanation of causality, he began shaping his own theory of absolute Forms: things are beautiful, because they partake of absolute Beauty in itself, and so on. At this stage of his philosophical development, he found his previous theory of comparison of magnitudes liable to paradoxes, such as the famous one:

"If the greater is greater and the less less by the head, then, first the greater is greater and the less less by the same thing; and secondly, the greater man is greater by the head which is itself small, and so you get the monstrous absurdity that a man is great by something small." (101 A-B)

The only way out from this "monstruous absurdity" is to refuse the proposition that "A is taller by a head than B, and B less by a head than A" and rather state that:

"the greater is greater *by*, and *by reason of*, *greatness*, and the less is less only *by*, and *by reason of*, *smallness*."

as well as that ten does not exceed eight by, and by reason, of two, but rather:

"by, and by reason of, *number*"

and that two cubits exceed one cubit not by a half, but by *magnitude* (*ibid.*)

I will deal with the philosophical relevance of this passage for the beginnings of a metatheory of comparatives in a future article. Here I limit myself to observe, first of all, that Plato's treatment of comparatives uses geometrical and arithmetical examples and a mathematical phraseology ("greatness", "smallness", "number", "magnitude"), and secondly, the fact that Plato's theory perfectly matches the idiosyncratic rule of presupposition introduced by the scholiast to explain Dionysius' homogeneity-condition: it is easy to presume that the scholiast derived it from Plato.

7.2. From a strictly logical viewpoint, Dionysius' definition does not appear "perversam" or simplistic or inaccurate any more; in the light of its derivation from a Euclidean conceptual frame and of its debt to Plato's theory of comparison sketched in the *Phaedo*, it reveals itself as not only coherent with some theoretical *arkhai*, both epistemological and onto-logical, of Greek culture, but also as a *sound* definition. In order to compare two objects in terms of more and less, it is necessary that the same quality or feature belongs to them both, i.e. that they partake of the same Form (in Platonic terms) and/or that they are two individuals falling under the same kind, that is to say that they are *homogeneous* (in logical, 'Aristotelean' terms). It would be out of order to compare Achilles with Tersites in terms of military courage or Penelope with Helen in terms of virtue. Logically speaking it would be absurd to say, e.g.: "This elephant is bigger than *this* ant", even if, pragmatically, it would not be so difficult to find, in *actual linguistic usage*, an appropriate context for this sentence: one has only to recall the many discussions about a feasible context for Chomsky's colourless green ideas!

8. I emphasise that this article is only meant to start an investigation into the "archaeology" of the Greek discourse on comparatives, both in the field of a grammatical *theory* and in that of a wider (mostly logical), *metatheory* of comparison, and that I wanted to open this program of research with a plausible hypothesis on the conceptual and textual derivation of Dionysius' definition of the ὁμοια συγκριτικόν.

Many points are yet to be investigated.

The second *diorismós* of the definition (the "one-to-many-hetero-

geneous" case) is still obscure to me, even if I think that it points to some rules of logic. The theory of comparison outlined in the *Phaedo* is to be considered as a first step only in Plato's meditation about this problem, which will reappear in other dialogues: it is not enough to say that two objects partake in Greatness in order to introduce a comparison between them, as this is only a presupposition; a comparison is only possible when we are aware of the two Forms (Metaforms?) of the More and of the Less, both deriving from the Principle of the Great-and-Small, which Plato introduces in the *Philebus*, which is to be considered one of the most important treatises for the development of a metatheory of comparatives. Let us recall that Apollonius Dyscolus (*synt.*, ed. Uhlig 1910:82, 9) will later state that comparatives incorporate "more" (ἐν τοῖς συγκριτικοῖς ἔγκειται τὸ πᾶλλον): this statement is not as obvious as it sounds to our linguistic consciousness as speakers of languages where the comparative is an analytical construction with the adverbs "more" and "less": *more difficult, plus difficile, più difficile* and so on. For a Greek speaker, whose comparatives are synthetic (*andreíoteros* = *more courageous*), the idea that this form incorporates "more" is not selfevident, but rather it requires an advanced and sophisticated theoretical insight, which only a philosophical as well as mathematical training could provide.

Another difficult point is to determine how grammarians could deal with cases diverging from the "one-to-one" condition, such as "Achilles is braver than Ajax and Agamemnon".

Last, but not least, it is necessary to support my hypothesis of a mathematical (geometrical) derivation of Dionysius' definition by an investigation of the epistemological prerequisites of Greek science, in order to show that such transposition of *arkhai* from one science to another was not only allowed, but was actually practiced in other instances.

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